Assign 7C TOPiC Based Practice

November 27, 2017 10:44 PM

Math Challengers Topic Based Practice
Probability, Counting & Combinatorics

Name:
Date:

1. Chun is playing a game in which he can score 0, 2, 6 or 10 points in each round. After four rounds the sum of his scores is 16. How many different scoring sequences could have produced this sum? (one such sequence to include is 0, 6, 10, 0).

(4) 10-2-2-2 10-6-0-0 (12) 6+4+12 = 22 ways.

(6) 6-6-2-2

2. In a school cafeteria, there are 2 different soups, 3 different main courses, and 3 different desserts. You are allowed to take at most one soup, at most one main course, and up to three desserts (but you cannot have two or more servings of the same dessert). How many different meals could you have? Include in your count the "meal" in which you eat nothing.

Soup MAIN DESS.
XAB XABC XABC
(3) × (4) AB AC BC
× ABC
(8)

12 × 8 = 96

3. How many subsets of 1, 2, 3, 4, 5, 6, have either 4, or 5 as their largest elements?

(1): 2 ways

(2): 41,42,43, 51,52,53,54 (7)

(3) $4C_3 + 5C_3 = 3+6$ (9)

(4) 4___ + 5___ : 1+4 (5)
$3C_3$ $4C_3$

(5) 5 4 3 2 1 (1)

2 + 7 + 9 + 5 + 1
24

4. Automibile licencse plates for a state consist of four letters followed by a dash and two single digits. how many different license plate combinations are possible if exactly one letter is repeated exactly once, but digits cannot be repeated?

A B C D - # #

$4C_2 \times 26 \times 25 \times 24 \times 10 \times 9$.

5. Lamant has a box containing only red marbles, blue marbles and green marbles. He needs to select at least 17 marbles without replacement to be sure at least one of them is green. He needs to select at least 18 marbles without replacement to be sure at least 1 of them is red. He needs to select at least 20 marbles without replacement to be sure all three colors appear among the marbles selected. How many marbles are in the box?"

RED: x
BLUE: y
GREEN: z

(1) x+y = 17-1 = 16
(2) y+z = 18-1 = 17
(3) x+z = 20-1 = 19

33
19

2x+2y+2z = 52
x+y+z = 26

6. You have a Magic Money Machine (MMM). Whenever you put in a penny, the MMM keeps the penny, but spits out either 5 or 8 pennies. So if you have only 1 penny, and use the MMM twice, you may end up with 9, 12, or 15 pennies. What is the largest number of pennies that it is impossible to end up with, if you have only 1 penny and are allowed to use the MMM as many times as you want? (Chicken McNugget Theorem)

GAINS of 4 or 7.

$4 \times 7 - 4 - 7$

$= 28 - 11 = 17$

7. Suppose that you play the following game: you toss a fair nickel, dime, and quarter at the same time. If you get at least one "head," stop (game over). If you don't, you toss the coins again. If you get at least one head, stop. Otherwise, go on . . . .When you toss for the last time, what is the probability that all three coins show heads? (conditional probability)

- ONLY WIN IF HHH.
- ONLY CONTINUE IF TTT.

$P(w) = \frac{a}{1-r}$

$\frac{1}{8} + \frac{1}{8}\left(\frac{1}{8}\right) + \frac{1}{8}\left(\frac{1}{8}\right)^2 + \frac{1}{8}\left(\frac{1}{8}\right)^3 + \cdots$

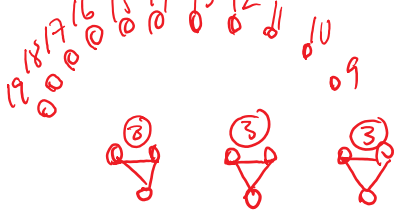
$= \frac{\frac{1}{8}}{1-\frac{1}{8}} = \frac{1}{7}$

8. How many ways can the numeral 20 be written as a sum of 3 distinctive positive integers?

17,2,1 16,3,1 15,4,1 15,3,2 14,5,1 14,4,2 13,6,1 13,5,2 13,4,3 12,7,1 12,6,2 12,5,3 11,8,1 11,7,2 11,6,3 11,5,4 10,9,1 10,8,2 10,7,3 10,6,4

20

9. Twenty people come to a party. Eleven of the people are friends with everyone else who came to the party. Also, the other nine people each have exactly thirteen friends at the party. (Assume that if A is a friend of B, then B is a friend of A.) Each person shakes hands with each of his / her friends. What is the total number of handshakes?



$\frac{11}{2}(9+19) + 9$

$\frac{11}{2}(28) + 9$

$11(14) + 9$

163

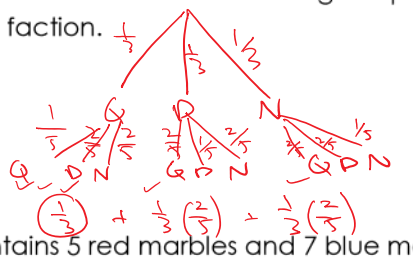
14
14
154
9

10. If three, standard, 6-faced dice are rolled, what is the probability that the sum of the face up integers is 16?

6,6,4 6,4,6 4,6,6 5,5,6 5,6,5 6,5,5

$\frac{6}{216} = \frac{1}{36}$

11. A driver approaches a toll booth and randomly selects two coins from his pocket. If the pocket contains 2 quarters, 2 dimes, and 2 nickels, what is the probability that the value of the two coins he selects will be at least enough to pay the 30-cent toll? Express your answer as a common faction.



$\frac{1}{3}+\frac{2}{15}+\frac{2}{15}$

$\frac{5}{15}+\frac{2}{15}+\frac{2}{15}$

$=\frac{9}{15}=\frac{3}{5}$

12. A Mayonnaise jar contains 5 red marbles and 7 blue marbles. A jelly jar contains 6 red marbles and 8 blue marbles. One marble is randomly selected from the mayonnaise jar and placed in the jelly jar. A marble is then selected from the jelly jar. What is the probability that the selected marble is red? Express your answer as a common fraction in lowest terms.

5R 7B 6R 8B

$\frac{7}{15}\left(\frac{5}{12}\right)+\frac{7}{12}\left(\frac{6}{15}\right)$

$\frac{35+42}{12(15)}=\frac{77}{180}$

13. What is the sum of all multiples of 13 up to 300?

$=13(0)+13(1)+\cdots\cdots\cdots 13(23)$

$=13(0+1+2+3\cdots\cdots 23)$

$=13\left(\frac{23\times24}{2}\right)=13\times23\times12=299\times12$

14. A palindrome is a number that reads the same forwards and backwards. What is the probability of picking a palindrome when picking a random 4-digit integer?

- 90 2 digit numbers
- $\frac{90}{9\times10\times10\times10}=\frac{1}{100}$

15. Cindy wishes to arrange her coins into X piles, each consisting of the same number of coins, Y. Each pile will have more than one coin and no pile will have all the coins. If there are 13 possible values for Y given all of the restrictions, what is the smallest number of coins she should have?

# piles	# coins
x	y
y	x
a	b
b	a
n	1
1	n

13 ways

15 ways

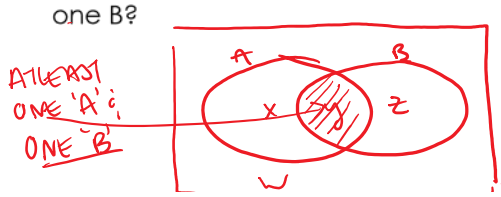
- smallest # 'N' with 15 FACTORS.

$N=a^4\times b^2$

$N=2^4\times3^2$

$=16\times9=144$

16. How many words of length 5 using the letters A, B, C, D, and E have at least one A and one B?



AT LEAST ONE 'A' & ONE 'B'

Total: $5^5=x+y+z+w$

No (A/B) $=3^5=w$

AT LEAST ONE A: $5^5-4^5=x+y$

$x+y+z+w-(x+y+z+w)=y$

$5^5-4^5+3^5-3^5=y$

$5^5-4^5-4^5+3^5=y$

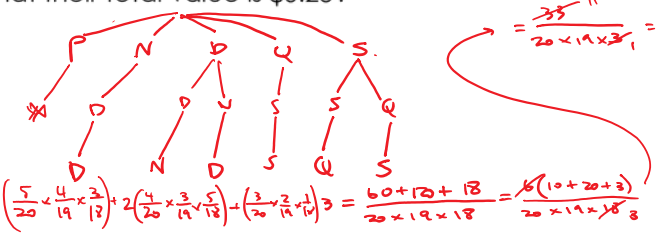
AT LEAST ONE A: $5^5 - 4^5 = x + y$

AT LEAST ONE B: $5^5 - 4^5 = z + y$

$5^5 - 4^5 - 4^5 + 3^5 = y$

17. A piggy bank contains six pennies, five nickels, four dimes, three quarters, and two worthless slugs. If the bank is shaken until three of the items drop out, what is the probability that their total value is \$0.25?

6P
5N
4D
3Q
2S



$\left(\frac{5}{20}\times\frac{4}{19}\times\frac{3}{18}\right)+2\left(\frac{4}{20}\times\frac{3}{19}\times\frac{5}{18}\right)+\left(\frac{3}{20}\times\frac{2}{19}\times\frac{1}{18}\right)3 = \frac{60+120+18}{20\times19\times18} = \frac{\cancel{6}(10+20+3)}{20\times19\times\cancel{18}_3}$

$= \frac{\cancel{33}^{11}}{20\times19\times\cancel{3}_1} = \frac{11}{380}$

18. The science club has 25 members: 10 boys and 15 girls. A 5-person committee is chosen at random. What is the probability that the committee has at least 1 boy and at least 1 girl?

$\frac{{}_{25}C_5}{{}_{25}C_5} - \frac{{}_{15}C_5}{{}_{25}C_5} - \frac{{}_{10}C_5}{{}_{25}C_5}$

$\frac{503}{506}$

19. There are ten girls and four boys in Mr. Middle's combinatorics class. In how many ways can these students sit around a circular table such that no boys are next to each other?

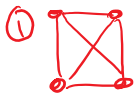
$\square G \square G \square G \square G \square G \square G \square G \square G \square G \square G \square G =$ $\frac{{}_{10}C_4 \times 4! \times 10!}{14}$

20. How many different ways are there to sit 15 people around a large dining table?

$\frac{15!}{15} = 14!$

21. There are four people in a room. For every two people, there is a 50% chance that they are friends. Two people are connected if:
    - They are friends
    - A third person is friends with both of them OR
    - They have different friends who are friends of each other

    What is the probability that every pair of people in this room is connected? Pascal #25



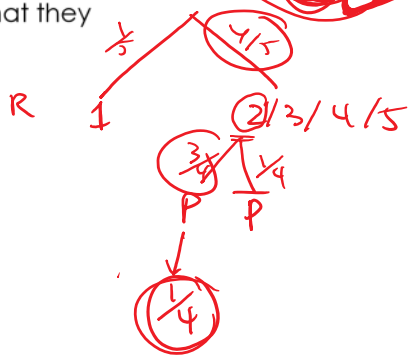
2^6: 64 ways to be connected

② ways to NOT connect.
- No lines → ①
- 1 line → ⑥
- 2 lines → $_6C_2 = 15$
- 3 lines (Terminus) ④

③ $64 - 26 = 38$

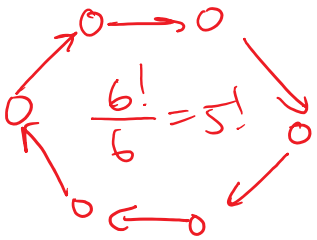
$P(x) = \frac{38}{64} = \frac{19}{32}$

22. Five monkeys are seated around a table. Their seats are labelled P, Q, R, S, and T, in clockwise order, as shown. The five monkeys are randomly numbered Monkey 1, Monkey 2, Monkey 3, Monkey 4, and Monkey 5. Monkey 1 will stay in its seat. The remaining four monkeys then sit themselves in the remaining seats so that they are seated in clockwise order as Monkey 1, Monkey 2, Monkey 3, Monkey 4, and then Monkey 5. What is the probability that they Monkey originally in seat R moves to seat P? Cayley #22



$\frac{3!}{4!}$

23. Six friends will exchange books in their book club. Each friend has one book to give to a friend, and will receive one book from a different friend. (No two friends trade books with each other). In how many ways can the books be exchanged? Cayley #24



$5! = 120$

10 × 4 = 40

160

Answer:

1 22 [0,0,10,6]→ 12
[10, 2,2,2]→ 4
[6,6,2,2]→ 6

2 96 [3x4x8]
3 – 2soups and none
4 – 3 soups and none
8 – 2x2x2 [y/n]
None is included

3 24 [4 biggest → 8 ways]
[5 biggest → 16 ways]

4 16,848,000

$\frac{4!}{2!}(26\times25\times24\times1)\times(10\times9)$

5 26
[B+R=16, B+G=17, R+G=19]

6 18 (chicken mcnugget)
$\#=a\times b-a-b$

7 1/7 [infinite geometric sequence] OR
7 choices, only 1 works

8 24
(stick and stone)
$\binom{k-1}{r-1}$

$\frac{(20-1)C_{3-2}-27}{3!}$
[27 doubles,
3! For repeats]

9 163 (combinatorics)

10 1/36 [just count]

11 3/5 [tree diagram]

12 $\frac{77}{180}$ [Tree diagram]

13 =12x13x23=3588
[arithmetic series]

14 1/100 [90/9000]

15 ANwer 144 {15 factors}
WrONG: 192= $2^6\times3^1$ [14 factors because 1 wasn't allowed]

16 ANSWER 1320 (good)
$5^5-4^5-4^5+3^5$
Use Venn Diagram
(Anna R.)
{{2882= 5^5-3^5 [find the complement]
This is wrong b/c there's also A's and no B's (Vice Versa)}}

17 $\frac{2C_2\times3C_1+4C_2\times4C_1}{19C_3}$
[Qxx, DDN]/19coins choose 3

18

$10C$	$15C$
1	4
2	3
3	2
4	1

Add them up

19 $10!\times11P_4$
[10 ways to seat the girls, then 11 spots for the boys]

20 14!

21 19/32 [count the complement 26 out of 64 ways that are disconnected]

22 3/20 (Use a tree

23 160 ways

diagram)